

Introduction

- This work develops a synergistic combination of Markovian and interval optimization for unit commitment problems with wind generation and transmission
- Motivation: Important to accommodate high penetration of wind
 - DOE's goal: **20%** wind by 2030
 - Obama's goal: **80%** clean energy by 2035
 - In Spain, an unprecedented decrease in wind generation in Feb. 2012 is equivalent to the sudden down of 6 nuclear plants (4 is not unusual)
 - Texas Emergency Electric Curtailment Plan is called on in Feb. 2008
- Difficulties:
 - Intermittent/uncertain nature of wind generation
 - Cannot be dispatched as conventional units
 - Large uncertainty: Mean Absolute Error (normalized over capacity) of day-ahead wind power forecast: 15%~20%
 - Complicated structures of transmission networks
 - Computational complexity: NP hard problems

Literature Review

- Stochastic programming
 - Modeling wind generation – Representative scenarios
 - To minimize the expected cost over scenarios
 - Difficult to choose an appropriate number of scenarios to balance computational complexity and solution feasibility
- Robust optimization
 - Uncertainties modeled by an uncertainty set w/o probabilities
 - To optimize against the worst-case realization
 - Min Max conservative and computationally challenging
- Pure interval optimization^[1]
 - Modeling wind generation – Closed intervals w/o probabilities
 - Capturing the bounds of uncertain inputs in different types of constraints, and making decisions feasible for these bounds
 - System demand constraints: As long as min. and max. wind realizations are feasible, other realizations within them will be feasible
 - E.g., wind farm 1 outputs [10 MW, 40 MW], and wind farm 2 [20 MW, 50 MW]. Total wind generation = [30 MW, 90 MW].
 - System demand = 200 MW. Net system demand = [110 MW, 170 MW]
 - If a set of committed units with p_i^{\min} and p_i^{\max} can meet the 110 MW and 170 MW, can it satisfy possible demand at 140 MW?
 - Transmission capacity constraints: $|\text{Power flow}| \leq f_l^{\max}$
 - A line flow is a linear combination of nodal injections weighted by **generation shift factors** (GSFs can be + or -)

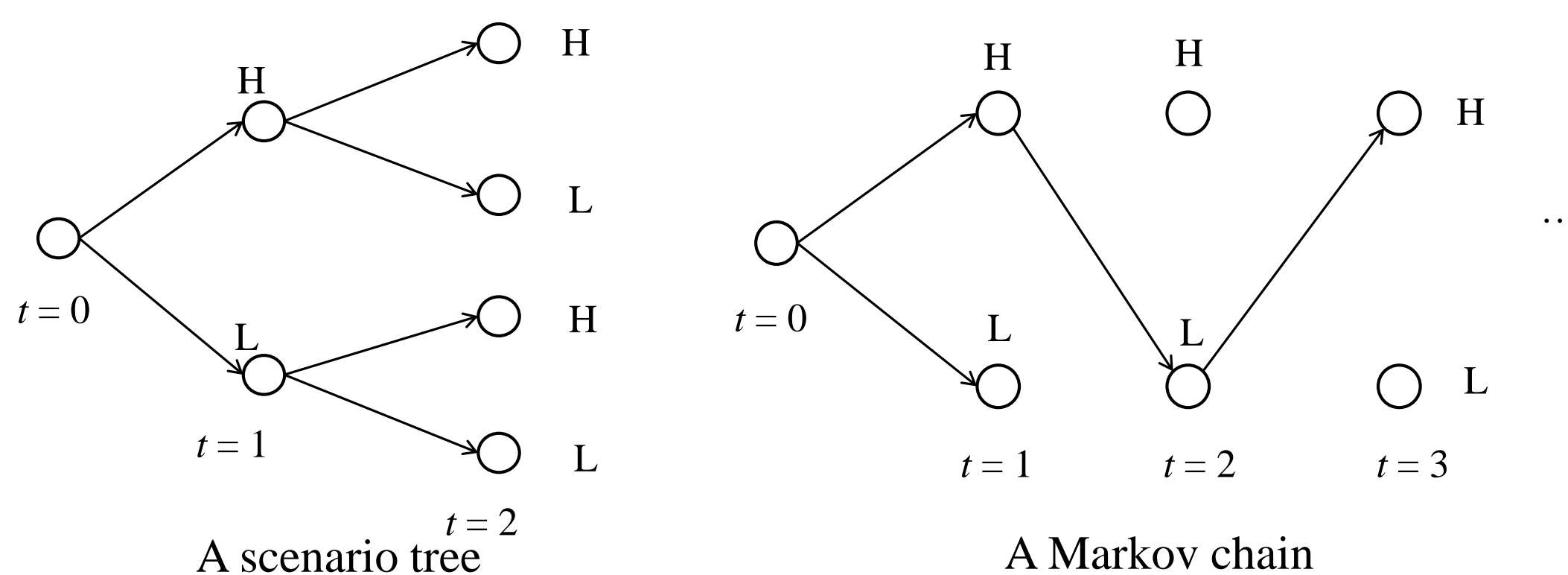
$$f_l(t) = \sum_i a_l^i \left(p_i^I(t) + p_i^W(t) - p_i^L(t) \right), \forall l, \forall t \quad (1)$$
 - “Passively” capture **bounds of uncertain inputs**

$$\sum_i a_l^i p_i(t) \leq f_l^{\max} - \max \left[\sum_i a_l^i \left(p_i^W(t) - p_i^L(t) \right) \right], \forall l, \forall t \quad (2)$$

Pre-computed based on interval arithmetic
 - Objective function: To minimize the cost of the expected realization
 - Linear and efficient via interval arithmetic; conservative

Previous Work - Markovian Optimization w/o Transmission^[2]

- Model aggregated wind generation – A Markov chain
 - Given the present, the future is independent of the past



N^T possible scenarios at one node

$T \cdot N$ possible states at one node

- Advantage:** State at a time instant summarizes the information of all previous instants in a probabilistic sense for reduced complexity

- Stochastic UC depends on **states instead of scenarios**

Markovian and Interval Unit Commitment

- Wind model considering transmission constraints**
 - With congestion**, wind generation cannot be aggregated together
 - Wind states for farms at different nodes may not be the same
 - Nearby wind farms: Generation aggregated
 - Wind farms far apart: States assumed independent
 - A Markov chain per node**
 - With I wind nodes (Markov chains): N^I possible global states at time t
 - Curse of dimensionality!**

Key idea: Markov + interval-based optimization

- Markovian analysis to depend on local states; interval analysis to manage extreme combinations of non-local states
 - Local state: Wind generation state at the node under consideration (will be extended into zonal state in future work)
- Physical infrastructure supporting this idea: **Wind-diesel system**
- How to combine two distinct approaches? Divide the generation (dispatch decision) of a conventional unit into two components**
 - Markovian component** depends on the local state n_i

$$x_i(t) p_i^{\min} \leq \boxed{p_{i,n_i}^M(t)} + \boxed{p_{i,\bar{n}_i}^I(t)} \leq x_i(t) p_i^{\max}, \forall i, \forall t, \forall n_i, \forall \bar{n}_i \quad (3)$$

- Interval component** manages extreme combinations of non-local states
- Constraints innovatively formulated to guarantee solution feasibility for all realizations without much complexity
- The effective use of local wind states alleviates the over-conservativeness of interval optimization

- System demand constraints
 - Based on interval optimization^[1]: **As long as min. and max. global states are feasible, all other realizations within them will be feasible**

$$\underbrace{\sum_i \left(p_{i,\min n_i}^M(t) + p_{i,m_i}^I(t) \right)}_{\text{The minimum local state at node } i} = \sum_j \left(p_i^L(t) - p_{i,\min n_i}^W(t) \right), \forall t \quad (4)$$

The minimum combination of non-local states (where other nodes are at their minimum possible states)

$$\sum_i \left(p_{i,\max n_i}^M(t) + p_{i,M_i}^I(t) \right) = \sum_j \left(p_i^L(t) - p_{i,\max n_i}^W(t) \right), \forall t \quad (5)$$

- Transmission capacity constraints: $|\text{Power flow}| \leq f_l^{\max}$
 - Flexibility of local conventional generation used to shrink ranges of RHS**

$$\sum_i a_l^i p_i^I(t) \leq f_l^{\max} - \max \left[\sum_i a_l^i \left(p_{i,n_i}^W(t) + p_{i,n_i}^M(t) - p_i^L(t) \right) \right], \forall l, \forall t \quad (6)$$

Markovian nodal injection $\equiv p_{i,n_i}^M(t)$ (containing decision variables)

- Ramp rate constraints
 - Required for possible local states, local state transitions, $p_{i,n_i}^I(t)$, and $p_{i,M_i}^I(t)$
- The objective function: To approximate the expected cost w/o much complexity
 - A weighted sum of extreme realizations and the expected realization

$$\min \sum_{t=1}^T \sum_{i=1}^I \left\{ \sum_{n_i=1}^{N_i} \left[w_{n_i,M_i}(t) C_i \left(p_{i,n_i}^M(t) + p_{i,n_i}^I(t) \right) + w_{n_i,M_i}(t) C_i \left(p_{i,n_i}^M(t) + p_{i,M_i}^I(t) \right) \right] \right\} \quad (7)$$

Weights adding up to 1
- A **non-linear** MIP formulation
 - Non-linearity lies in max/min (negative flow direction) operations in (6)

Solution Methodology – Branch-and-cut

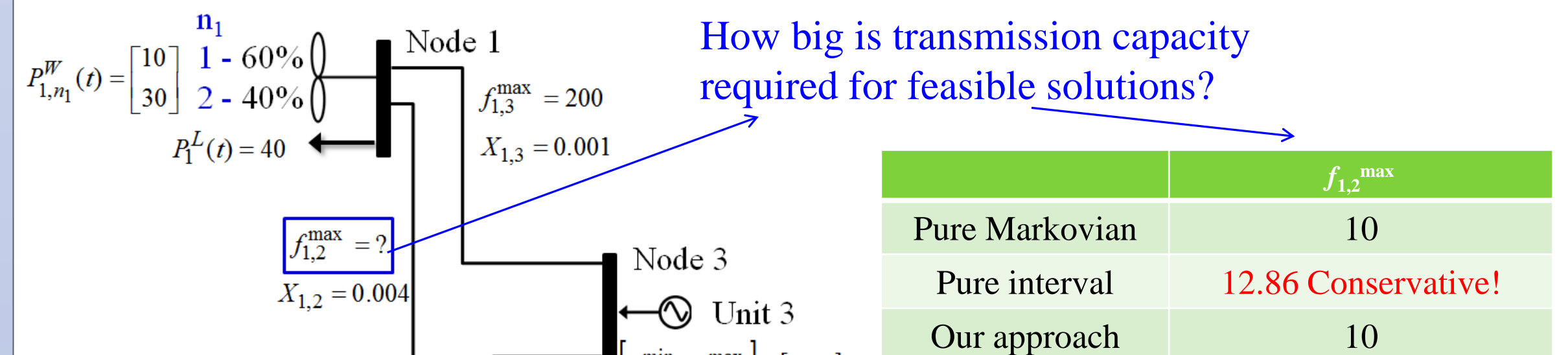
- Max/Min operations transformed into a linear form**
 - Idea:** Analyze the monotonicity of Markovian nodal injections w.r.t. local states, then **select indices of local states w/o optimization**
 - The Monotonicity Conjecture:** The local state with lower wind generation provides less or equal Markovian nodal injection at the optimum, i.e.,

$$p_{i,n_i-1}^M(t) \leq p_{i,n_i}^M(t), \forall i, \forall t, \forall n_i, \forall (n_i-1) \in \{n_i-1 \mid \varphi_{n_i-1}(t) > 0\}. \quad (8)$$
 - Generalized monotonicity analysis** used to support this conjecture

$$\max_{n_i} p_{i,n_i}^M(t) = p_{i,\max n_i}^M(t), \quad \min_{n_i} p_{i,n_i}^M(t) = p_{i,\min n_i}^M(t), \forall i, \forall t. \quad (9)$$
 - Overall problem converted linearly after
 - Including (8) as constraints
 - Substituting the min/max operations with corresponding states
- State transition matrices given and state probabilities pre-computed**

Numerical Testing Results

- CPLEX 12.5.1.0 on a PC laptop with an Intel Core(TM) i7-2820QM 2.30GHz CPU and 8GB memory
- Illustrative examples
 - Conservativeness** – Consider 3 nodes, 2 wind farms, 2 units, and 1 hour



	No. of dispatch decisions per unit	No. of flow levels per line
Pure Markovian	10⁶	10⁶
Pure interval	2+1	2+1
Our approach	10 + 2 + 1	2 + 2 + 1

- Complexity** – Consider 6 wind farms at different buses, 10 states for each
- Solution feasibility and modeling accuracy**
 - IEEE 30-bus system with 2 wind farms at 40% wind penetration
 - Free wind curtailment and load shedding at \$5,000/MWh penalty
 - Stopping MIP gap 0.1% and then 10,000 Monte Carlo runs
 - Our approach provides 5.23% lower simulation cost** than pure interval
 - Our approach is the most accurate, as it has the smallest APE[#]**

Approach	Deter.	Interval	Ours
Optimization	CPU time	2s	53s
	Cost (k\$)	248.66	280.67
	Penalty (k\$)	0	0.47
UC cost (k\$)		89.46	67.72
Simulation	E(Cost) (k\$)	314.89	263.26
	APE [#]	21.03%	6.61%
	STD(cost) (k\$)	74.46	33.77
	Penalty (k\$)	40.82	0

[#] Absolute percentage error (APE) = |optimization cost – simulation cost| / simulation cost × 100%

- Computational efficiency**

- IEEE 118-bus system with 3 wind farms

		Ours
Optimization	CPU time	41s
	MIP GAP	0.01%
	Cost (k\$)	911.48
UC Cost (k\$)		12.83
Simulation	E(cost) (k\$)	920.97
	APE	1.03%
	STD(cost) (k\$)	24.64

Conclusion

- An important but difficult issue
- Hybrid Markovian and interval optimization to overcome the complexity caused by transmission constraints
 - Markovian analysis to depend on local state/reduce conservativeness
 - Interval analysis to ensure feasibility against realizations
- Problem transformed into a linear form based on monotonicity, and then solved efficiently by using branch-and-cut
- Opens a new and effective way to address stochastic problems w/o scenario analysis and avoid over-conservativeness

References

- Y. Wang, Q. Xia, and C. Kang, “Unit commitment with volatile node injections by using interval optimization,” *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1705-1713, 2011.
- P. B. Luh, Y. Yu, B. Zhang, E. Litvinov, T. Zheng, F. Zhao, J. Zhao, and C. Wang, “Grid integration of intermittent wind generation: A Markovian approach,” *IEEE Trans. Smart Grid*, vol.5, no.2, pp.732-741, March 2014.
- Y. Yu, P. B. Luh, E. Litvinov, T. Zheng, F. Zhao, and J. Zhao, “Grid integration of distributed wind generation: A Markovian and interval approach,” submitted.

Thank you!